Abstract

The reconstruction of a low rank matrix, given only a small subset of its entries, has recently gained a lot of importance with the introduction of Netflix problem. Under no other assumption, the problem seems impossible because each such missing entry can take any value on the real number line R[[1]](#footnote-1)which make an infinite matrices on , all equally probable candidates for original matrix. So the problem was rather an ill-posed one, only after Candes and Retch in 2008 proved mathematically that in case of nxn matrix[[2]](#footnote-2), we require only entries to recover the whole matrix with high probability, given only the assumption that the matrix is low rank and possesses certain properties so called weak coherence property. Interesting point is that in practical scenarios a majority of matrices satisfy these expectations and are subject to exact recovery, even if a small number of entries are revealed. This problem is of considerable practical interest as in most of practical scenarios having large data matrices, there are only a few independent directions that can actually represent the whole matrix. So, often there is either a low rank or an approximately low rank structure lying hidden in that higher dimensional space. Some of many examples include Netflix prize problem [1] movie-user data matrix, sensor net distance matrix and quantum tomography [2].

Apart from theoretical guarantees Candes and Recht has also shown that the unknown matrix X can be obtained from the partially revealed matrix by solving a unique convex optimisation problem called the nuclear norm minimisation. Nuclear norm minimisation introduced by Fazel in her Phd thesis is the tightest convex relaxation of the rank minimisation problem which is the solution to matrix completion problem. This is similar to using the popular l1 norm as a proxy for l0 norm in compressed sensing and medical imaging literature. The idea is that in both the cases the former is computationally more tractable than the latter. The dual of above nuclear norm minimisation can be casted into a semi definite program which can be solved by SDP solvers. However SDP solvers aren’t very efficient for large problems so different research groups have been proposing more efficient, but often limited in scope, alternative algorithms for solving matrix completion problem. These include low rank matrix factorization by Srebro etal and Monteiro etal, FPCA (Fixed Point Continuation with approximation SVD) by Chen etal, SVT (Singular Value Thresholding) by Candes etal and OptSapce by Keshavan and Oh. The further study on matrix completion problem has taken two routes, one improving on theory for better and closer results the other finding and improving new algorithms which can converge fast enough and solve a broader domain of large practical problem.

1. Complex number are also allowed, however without losing generality reals are taken for simplicity. [↑](#footnote-ref-1)
2. Rectangular matrices are also allowed but square ones are simple to consider for analysis. [↑](#footnote-ref-2)